ONE EXAMPLE OF TURBULENT FLOW WITH A STRONG EFFECT ON THE PULSATION CHARACTERISTICS OF THE FLOW

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A semiempirical theory of turbulent transfer is used to calculate the flow of a liquid in a tube containing macroscopic formations. The properties of the solution correspond with the data on the effect of polymer additives on the turbulent flow characteristics.

1. In Hoit's review [1], the possibility is mentioned that molecules of a polymer, being passive and present in the flow, interact mechanically with turbulent pulsations.

In this paper, an attempt is made to construct a scheme of interaction of turbulent flow with the macroscopic formations present in the flow. The scheme is based on the representation of the existence in solution of relatively coarse formations and associates, which was developed in [2-4]. In a current with sufficient shear stress, these macroformations are drawn along with the flow. We shall start from the assumption that the macroformations are filiform cylindrical fibers, of neutral buoyancy and with length L_S and radius of cross-section L_0, not very rigid in their structure (gelatinous). The density of the fiber is assumed equal to the density of the surrounding medium. Their quantity in the stream is defined by the concentration n_0.

2. Turbulent flow is a collection of vortices, nodules of liquid with correlated velocities, molar diameter 2L, which are generated at every point of the flow and are transported while interacting with the surrounding medium. The velocity distribution is described by the probability density of the distribution function f, which satisfies the equation [6, 7]:

$$\frac{\partial}{\partial x_K}(u_K f) + \frac{\partial}{\partial u_K}(F_K f) = \frac{f_0 - f}{\tau},$$
$$\tau = ALE^{-1/2},\quad E = \frac{1}{2}\langle u_K'^2 \rangle, \tag{1}$$

f_0 is a normal distribution function with local parameters $\langle u_K \rangle$ and E. In a stream without admixtures,

$$F_K \cong -\frac{3}{8} a_0 A u_K'/\tau.$$

The transport equation follows from Eq. (1)

$$\frac{\partial}{\partial x_K}\langle u_K Q \rangle - \left\langle F_K \frac{\partial Q}{\partial u_K} \right\rangle = \int \tau^{-1}(f_0 - f)\, Q\, du_x du_y du_z, \tag{2}$$

which gives the corresponding momenta equations. For flow in a layer of constant friction stress, the solution has the form [7] (additionally, the equation for the scale L is used):

$$\langle u_x'^2 \rangle_0^+ = \frac{2}{3} E_0^+ \frac{1 + \frac{9}{4} a_0 A}{1 + \frac{3}{4} a_0 A},\quad \langle u_y'^2 \rangle_0^+ = \frac{2}{3} E_0^+ \frac{1}{1 + \frac{3}{4} a_0 A},$$

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$$E_0^+ = \left(1 + \frac{3}{4} a_0 A\right) [2(a_0 A)^{-1}]^{1/2}; \; L^+ \frac{dU^+}{dy^+} = \frac{3a_0}{4} (E_0^+)^{3/2}, \; L^+ = 0.4y^+. \quad (3)$$

The values of the constants a_0 and A are chosen from comparison with experimental data, $3a_0A = 4$ and A = 3.86.

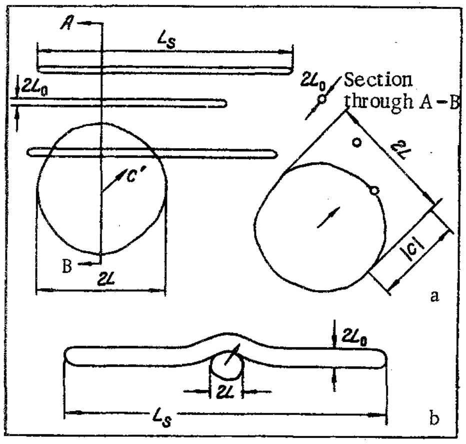

Fig. 1. Explanation of the scheme for the interaction between vortex and fiber.

3. In a current with distributed macroscopic fibers, an additional resistance for a moving vortex will originate when it interacts with the fibers. The mechanism for the supression of velocity pulsations due to interaction with the fibers was discussed qualitatively in the papers of Van-Driest (the interaction of turbulent pulsations with molecule-rods is proportional to the velocity u') [8] and Black (discrete dynamic interaction between macromolecules and small scale turbulence in the vicinity of a wall) [9] and in [5], etc.

We shall describe the interaction in the following way. The fibers are drawn through the stream (Fig. 1). A vortex, while moving in the stream, after unit time encounters a specified number of fibers. It is equal to the product of the surface area swept by the vortex in the transverse section $2L|c|$ and the fiber concentration n_1 in the transverse section (the fibers are long). The quantity n_1 is numerically equal to n_0. With each interaction between the fiber and the vortex, the fiber transfers a certain momentum, which also amounts to an additional resistance acting on the vortex.

When the size of the vortex is large (compared with the length of the fiber), the mass of the vortex is considerably greater than the mass of the fiber and it may be drawn almost entirely into motion. The momentum transferred can be assumed to be proportional to the quantity u'M, where $M = \rho\pi L_0^2 L_S$ is the mass of the fiber.

When the size of the vortex is quite small ($L \le L_0 \ll L_S$), the mass of the vortex is considerably less than the mass of the fiber and the vortex can draw into motion only a part of the fiber, to the extent of length ~L, then the momentum transferred can be assumed to be proportional to the quantity $u'ML/L_S$. The interpolated formula

$$u'M\left(\frac{L_s}{K_1 L} + \frac{1}{K}\right)^{-1} = \frac{u'ML_0}{L_s}\varkappa\varphi$$

is used below. The additional drag force acting on unit mass of vortex has the form

$$\Delta F_K \cong -\frac{u'_K \varepsilon}{2\tau},$$

$$\varepsilon = \frac{E\varphi}{y^+}, \quad E = \frac{3A}{\pi} c_1 L_0^+, \quad c_1 = n_1 \pi L_0^2. \quad (4)$$

4. Using relation (4), we have

$$F_K = -\frac{u'_K}{2\tau}\left(\frac{3}{4} a_0 A + \varepsilon\right).$$

Substituting this expression in the transport equation (2), we obtain approximate momenta equations for the friction in a layer of constant turbulent friction stress (we shall neglect diffusion transfer of the energy of the pulsation motion):

$$2\langle u'_x u'_y\rangle \frac{dU}{dy} = -\frac{\langle u'^2_x\rangle}{\tau}\left(\frac{3}{4} a_0 A + \varepsilon\right) + \frac{1}{\tau}\left(\frac{2}{3}E - \langle u'^2_x\rangle\right),$$

$$0 = -\frac{\langle u'^2_y\rangle}{\tau}\left(\frac{3}{4} a_0 A + \varepsilon\right) + \frac{1}{\tau}\left(\frac{2}{3}E - \langle u'^2_y\rangle\right),$$

$$2\langle u'_x u'_y\rangle \frac{dU}{dy} = -\frac{2E}{\tau}\left(\frac{3}{4} a_0 A + \varepsilon\right),$$

$$\langle u'^2_y\rangle \frac{dU}{dy} = -\frac{\langle u'_x u'_y\rangle}{\tau}\left(1 + \frac{3}{4} a_0 A + \varepsilon\right). \quad (5)$$

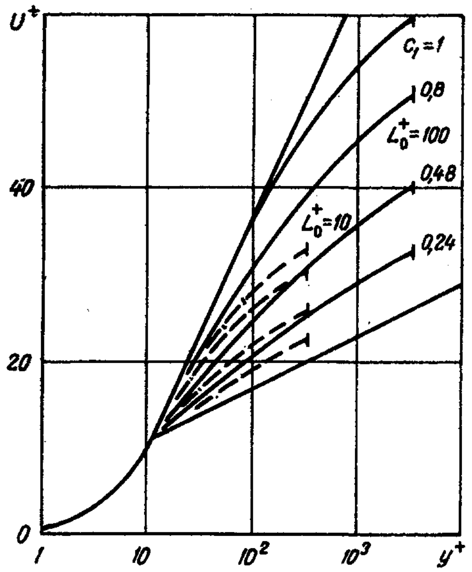

Fig. 2

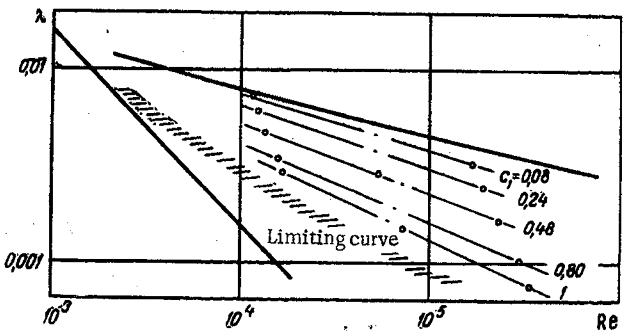

Fig. 3

Fig. 2. Velocity profile versus fiber concentration, for different values of the Reynolds number.

Fig. 3. Dependence of the drag coefficient for flow in a tube on the Reynolds number and the fiber concentration.

The relation $L = \varkappa y$ is used for the scale, where $\varkappa = 0.4$. The solution of this system of equations has the form

$$E^+ = E_0^+\left(1+\frac{\varepsilon}{2}\right)(1+\varepsilon)^{-1/2};\quad \langle u_x'^2\rangle^+ = \langle u_x'^2\rangle_0^+\left(1+\frac{3}{4}\varepsilon\right)(1+\varepsilon)^{-1/2};$$

$$\langle u_y'^2\rangle^+ = \langle u_y'^2\rangle_0^+(1+\varepsilon)^{-1/2};\quad L^+\frac{dU^+}{dy^+} = (1+\varepsilon)^{1/4}\left(1+\frac{\varepsilon}{2}\right)^{3/2}. \tag{6}$$

When the solution is extended to the wall, a two-layer scheme is assumed: for $y^+ < 11.3$, the velocity $U^+ = y^+$, and for $y^+ > 11.3$, the formulas (6) are used. Here, we take $3a_0A = 4$, $|\tau_W/\rho| = V_*^2$, $U^+ = U/V_*$, $y^+ = yV_*/\nu$ and $\lambda = 2(U_1^+)^{-2}$ is the drag coefficient of the tube, U_1 is the mean value of the flow velocity in the tube. The solution depends on a number of parameters:

$$L_0^+ = (L_0/D)\,\mathrm{Re}\,\lambda^{1/2} = (L_0/D)\,2.82R^+,$$

E (for a given value of L_0^+ defines the magnitude of the concentration c_1), K_1, L_0/D, $(\varkappa K_1/K)\,L_0/L_S = P$. In the example quoted, we took

$$K_1 = 0.67;\ P = 0.2;\ L_0/D = 10^{-2}.$$

With the assumed numerical values of these parameters, the solution depends on L_0^+ and c_1. The velocity profile $u^+(y^+)$ and the dependence for the drag coefficient on the Reynolds number for different concentration values are shown in Fig. 2 and 3.

The solution is conducted in the following way:

1. The velocity profile starts to distort in the vicinity of the boundary with a viscous sublayer and away from the wall it tends to transform to a logarithmic profile with a normal slope. The transition zone increases with increase of the fiber concentration value. The value of the constant K_1 is chosen such that, when $c_1 = 1$, a "limiting" profile was achieved, close to that observed experimentally.

2. The drag coefficient decreases with increase of concentration. In the formal solution, the magnitude of the concentration attains a maximum value when the fibers densely adjoin one to the other. This means, that the nature of the flow must be reorganized, as the interaction of the fibers becomes more complex. Qualitatively, this can be compared with the existing data on the effect of volume concentration

on the decrease of drag, the existence of a "critical concentration" and a limit in the decrease of the drag with increase of the concentration (references in [1] and [10]). Figure 3 shows the relation for different values of c_1 and the limiting experimental curve.

The solution contains the effect of the tube diameter. For a given Reynolds number, an increase of the tube diameter corresponds to a reduction of concentration. A reduction in the value of the fiber length L_S with a fixed transverse size leads to a reduction of the effect of decreasing the drag.

The general nature of distortion of the velocity profile conforms with the experimental data of [11, 15] etc. (references in [1, 8, 15]).

3. The relations (6), following from the solution, also conform qualitatively with the experimental data of [12] and the data of [13, 14, 15]: the intensity of the transverse velocity component of the pulsation motion falls, the intensity of the longitudinal component increases and the anisotropy in the distribution of the velocity pulsation components increases. The maximum value of the quantity ε, attained at the boundary with the viscous sublayer, is equal to 2.5.

Thus, the application of the phenomenological theory of the mixing length, taking into account the dynamic interaction of the vortices with passive macroformations, leads to a number of inferences which conform with the observed mechanisms for reducing drag in a turbulent stream of a polymer solution. It can be seen that the pattern of interaction between the vortices and fibers is more complex because of the possible anisotropy of the force effect, due to the viscoelastic properties of the molecules. But it appears that the hydromechanical approach also merits attention.

NOTATION

f	is the function of the velocity distribution in turbulent flow;
τ	is the correlation time;
L	is the mean integrated scale of turbulence;
u'_K	is the pulsation components of velocity;
E	is the mean value of pulsation energy;
$\lvert c \rvert$	is the modulus of velocity of pulsation motion;
a_0, A	are the empirical constants;
F_K	are the components of the force of interaction between a vortex and the surrounding medium;
L_S	is the length of a fiber;
L_0	is the radius of fiber transverse section;
n_1	is the concentration of fibers in the transverse section of the stream;
c_1	is the volume concentration of fibers;
M	is the mass of a fiber;
m	is the mass of a vortex;
τ_W	is the frictional force;
V_*	is the frictional velocity;
λ	is the drag coefficient of tube;
U_1	is the mean value of flow velocity in tube;
D	is the diameter of tube;
φ	is the interpolation function, assumed for describing the interaction between vortex and fiber;

$\varepsilon = E\varphi/y^+$;
$E = (3A/\pi)c_1L_0^+$;
$c_1 = n_1\pi L_0^2$;
$U^+ = U/V_*$;
$y^+ = yV_*/\nu$;
$Re = U_1D/\nu$;
$D = 2R$;
$\lambda = 2\lvert\tau_W\rvert/\rho U_1^2 = 2/(U_1^+)^2$.

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